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PROBABILITY OF FAILURE AND RISK ASSESSMENT  
OF PROPULSION STRUCTURAL COMPONENTS

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## ABSTRACT

Due to increasing need to account for the uncertainties in material properties, loading conditions, geometry, etc., a methodology has been developed to determine structural reliability and to assess the risk associated with it. The methodology consists of a probabilistic structural analysis by a probabilistic finite element computer code NESSUS (Nonlinear Evaluation of Stochastic Structures Under Stress), and a generic probabilistic material properties model. The methodology is versatile and is equally applicable to high and cryogenic temperature structures. Results obtained demonstrate that the whole issue of structural reliability and risk can be formally evaluated using the methodology developed which is inclusive of uncertainties in material properties, structural parameters and loading conditions. The methodology is described in some detail with illustrative examples.

## INTRODUCTION

The probabilistic structural analysis method (PSAM)<sup>1</sup> has been developed to analyze the effects of fluctuating loads, variable material properties, and uncertainties in analytical models, especially for high performance structures such as SSME turbopump blades. In the deterministic approach, uncertainties in the responses are not quantified and the actual safety margin remains unknown. Risk is calculated after extensive service experience. However, probabilistic structural analysis provides a rational alternative method to quantify uncertainties in structural performance and durability. NESSUS is a probabilistic structural analysis computer code which integrates finite element methods and reliability algorithms,<sup>2,3</sup> capable of predicting the scatters of structural response variables such as stress, displacement, natural frequencies, buckling loads, etc. These are subsequently compared with their probable failure modes to assess the risk of component fracture. Probable failure modes are included for different structures and their respective service environments. For example, failure events such as stress greater than strength, displacements exceed maximum allowables or avoidance of resonance are often used for the reliability assessment. Probability of occurrence of those failure events can be determined once the probability distributions of the requisite structural response variables are calculated by NESSUS.

In undertaking a reliability/risk analysis, all suspected sources in uncertainties must be taken into account in order to control the probability of failure in service environments within an acceptable range. Structural reliability and risk obtained by a formal probabilistic methodology can be useful in evaluating the traditional design, setting quality control requirements, inspection intervals and retirement for cause. It can also be used to identify candidate material and design concepts in the absence of a technology base. In this report, the methodology developed to assess the structural reliability and risk/cost is described.

## CONCEPT OF PROBABILISTIC STRUCTURAL ANALYSIS

In a probabilistic structural analysis, the primitive variables which define the structure have to be identified. These include temperature, material properties, structural geometry, loading conditions etc. and should be described by their respective probability distributions. A structural analysis performed by NESSUS with the pre-determined probability distributions of all the primitive (random) variables will produce corresponding scatter (uncertainties) in the structural responses such as displacement, stress, natural frequencies etc. The concept is illustrated by Figure (1) where the structural model synthesizes the input uncertainties.

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## STRUCTURE OF NESSUS

NESSUS consists of 3 major modules; NESSUS/PRE, NESSUS/FEM, and NESSUS/FPI.

NESSUS/PRE is a pre-processor used for the preparation of the statistical data needed to perform the probabilistic finite element analysis. It allows users to describe the uncertainties in the structural parameters (random variables) at the nodal points of a finite element mesh. The uncertainties in these parameters are specified over this mesh by defining the mean value and standard deviation of the random variable at each point, together with an appropriate form of correlation. Correlated random variables are decomposed into a set of uncorrelated vectors by a modal analysis. For strong correlation problems, the number of dominant random variables in the set of uncorrelated vectors will be much less than that of correlated random variables. The computational time required for the analysis will also be reduced significantly.

NESSUS/FEM is a finite element code used for the structural analysis and parameter sensitivity evaluation. It generates a database containing all the response information corresponding to a small variation of each independent random variable. The algorithm used in NESSUS/FPI requires an explicit response function in order to perform a reliability analysis. In complicated structural analysis problems, response can only be available implicitly through a finite element model. To overcome this difficulty, the response function is expressed parametrically with this database.

NESSUS/FPI (Fast Probability Integrator) is an advance reliability method<sup>3</sup>. This module extracts the database generated by NESSUS/FEM to develop a response or a performance model in terms of uncorrelated random variables. The probabilistic structural response is calculated from the performance model. For a given response value, the probability of exceedence at this value is estimated by a reliability method, which treats the problem as a constrained minimization. This step is called a point probability estimation. The cumulative distribution function is generated by running FPI at several response values. One alternative for generating the distribution function for any given response is to conduct a direct Monte Carlo simulation study. However, in general, it is very costly. NESSUS/FPI provides a method which not only produces a reliable distribution, but also requires less computing time than that for Monte Carlo simulation, especially in low probability regions.

## PROBABILISTIC MATERIAL PROPERTIES MODEL

A generic material behavior model<sup>4</sup> is used to evaluate the scatters of material properties for the structures subjected to high temperature environments and high cycle loading conditions. The fundamental assumption for this model is that the material properties behavior can be simulated by primitive (random) variables. The general form of this model is shown in TABLE (1). The exponents  $n$ ,  $p$ ,  $q$  are determined from available experiment data or can be estimated from the anticipated material behavior due to the particular primitive random variable.

## APPLICATION TO A SPACE SHUTTLE MAIN ENGINE (SSME) TURBINE BLADE

The methodology described previously will be applied to SSME turbine blade as shown in Figure (2). This blade is subjected to complex mechanical and thermal loads<sup>5</sup>; centrifugal force, thermal loads and the differential pressure across the airfoil. Centrifugal force is induced by rotational speed. Since it is difficult to maintain a constant rotational speed, the centrifugal force has to be considered as a random variable. Random thermal loads are due to combustion irregularities which causes a random temperature distribution in the blade. Differential pressure is also random because of pressure fluctuation. Uncertainties in the blade geometry arise during the manufacturing process. The scatter in material properties is caused by nonuniformities in the material.

The turbine blade is modelled by 40 4-node shell elements with 55 nodal points. In this study, 7 random fields are considered as listed in TABLE (2). In previous analysis,<sup>6</sup> the probability density function of material properties such as Young's modulus, thermal expansion coefficient and material strength are assumed. In the present study, they are simulated by the probabilistic material property model as defined in the TABLE (1). The material property is a function of its reference value, temperature, and fatigue cycles. The statistics of the primitive random variables are listed in TABLE (3). Since the material is a function of stress and stress is a function of material property in an implicit way, an iterative procedure is necessary to obtain a convergent solution for stress and material properties. The procedure is illustrated in TABLE (4). Young's modulus, thermal expansion coefficient and material strength are considered to be random variables. During the iteration process, the joint cumulative distribution functions of nodal stresses are also calculated in order to apply the generic probabilistic material property model on the entire blade. It is found that only a few iterations are needed for a convergence. The blade is also assumed to be subjected to 100,000 constant amplitude load cycles which degrade the modulus and strength. The critical points where the large displacement or high stress located are depicted in Figure (3). As shown in Figure (4), at the root of the leading edge, the probability distribution of modulus is reduced significantly after the application of cyclic loads. Changes in the probability density function of tip displacement

with and without the application of cyclic loads are also shown in Figure (5). As expected, the tip displacements are increased after the application of cyclic loads because the blade becomes softer. Material strength degradation after cyclic loads is calculated with the converged stress by the probabilistic material property model as shown in Figure (6). Varying the number of cycles and repeating the procedure described previously, stress strength relationships are determined for a given cycle. These relationships are used to define a failure mode as the event when the stress is greater than the corresponding material strength. The probability of failure is then calculated by

$$P_f = \int_{-\infty}^{+\infty} \left( \int_{-\infty}^x f_S(s) ds \right) f_\sigma(x) dx \quad (1)$$

where  $f_\sigma$  is the probability density function (pdf) of effective stress calculated by the probabilistic structural analysis using NESSUS.  $f_S$  is the pdf of material strength simulated by the probabilistic material property model. From this analysis, a Risk-Fatigue cycle curve is developed for critical locations as shown in Figure (7). This curve is useful for assessing the risk of structural fracture. For instance, at a given acceptable risk level, the number of fatigue cycles to initial local failure can be determined. With this information available, criteria can be set for quality control, inspection intervals and retirement for cause.

The risk assessment includes the initial cost and the consequential cost. The initial cost is defined as the cost for component service readiness which can be a function of several key design variables. The consequential cost is the cost incurred due to failure. Total cost is the sum of initial cost and a fraction of consequential cost as defined in Equation (2). The fraction is weighted by the probability of failure.

$$C_t = C_i + C_f \quad (2)$$

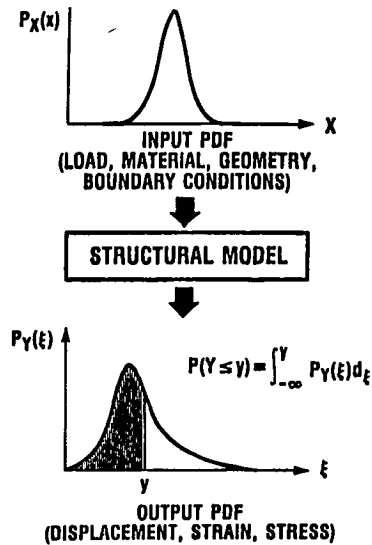
where  $C_i$  represented initial cost and  $C_f$  is the consequential cost if failure occurs. Since the lower initial cost is often associated with higher risk for the structural failure and higher initial cost will normally reduce the risk, the total cost can be minimized for an acceptable structural reliability. For example in Figure (8), the initial cost is a function of the quality of the material; more cost is needed for improved quality, yet the cost due to failure is reduced. There exists a point of diminishing return. This can be identified as the lowest total cost but with no loss of the structural reliability. A similar result can be obtained for the case of mean strength improvement as shown in Figure (9).

## SUMMARY

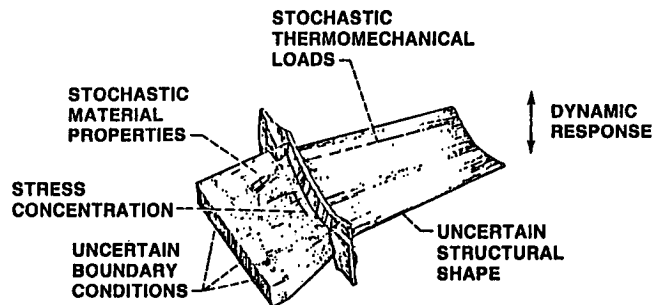
In summary, a reliability/risk cost methodology has been developed. It consists of a probabilistic structural analysis by NESSUS and a generic probabilistic material model. The methodology is versatile and equally applicable to hot and cold structures where data is difficult to obtain. The methodology is demonstrated by using it to assess the risk associated with fatigue cycles to initiate local failure in SSME blades.

## REFERENCES

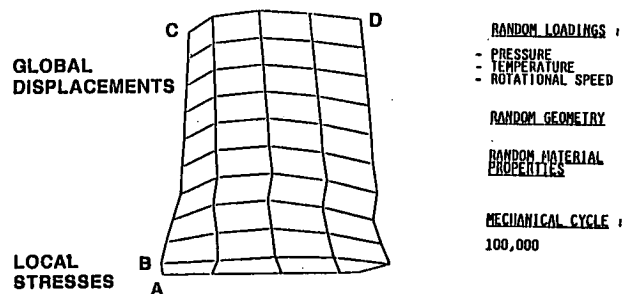
- (1) Chamis C. C. "Probabilistic Structural Analysis Methods for Space Propulsion System components", presented in 3rd Space System Technology AIAA Conference, San Diego, California, June, 1986.
- (2) Lind, N. C., Krenk, D., Madsen, H. O., "Safety of Structures", Prentice-Hall, 1985.
- (3) Wu, Y. T., "Demonstration of a New, Fast Probability Integration Method for Reliability Analysis", Proceedings of Symposium on Probability Structural Design and analysis, ASME, 1985.
- (4) Boyce, L., Chamis, C. C., "Probabilistic Constitutive Relationships for Cyclic Material Strength Models", 29th SDM Conference, Williamsburg, Virginia, 1988.
- (5) "Probabilistic Structural Analysis for Select Space Propulsion System Components", Annual Report by Southwest Research Institute, Rocketdyne, University of Arizona, Marc Analysis Corporation and Columbia University for NASA Lewis Research Center, Contract NAS3-24389
- (6) Shiao, M., Vinod, K. N., Chamis, C. C., "Probabilistic Structural Analysis of Aerospace Components Using NESSUS", 29th SDM Conference, Williamsburg, Virginia, 1988.



**FIGURE (1) CONCEPT OF PROBABILISTIC STRUCTURAL ANALYSIS**



**FIGURE (2) UNCERTAINTIES IN THE PROBABILISTIC STRUCTURAL ANALYSIS**



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**FIGURE (3) SSME BLADE SHOWING LOCATIONS WHERE PROBABILISTIC STRUCTURAL RESPONSE WAS EVALUATED**

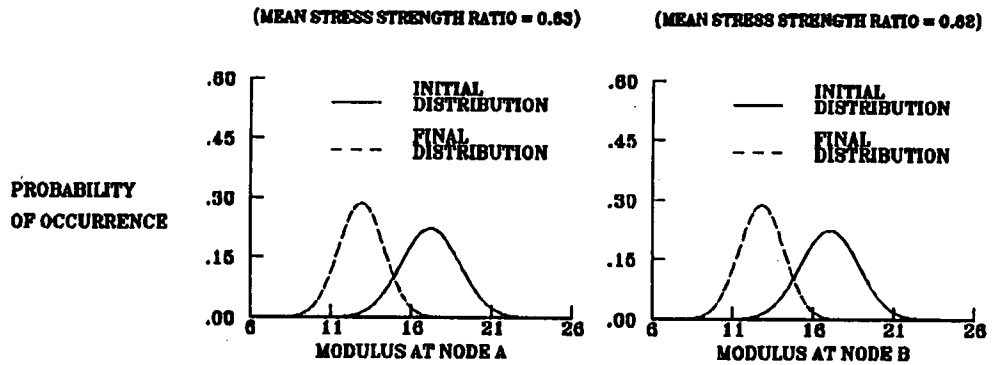


FIGURE (4) PROBABILISTIC MODULUS SIMULATED BY USING THE GENERIC PROBABILISTIC MATERIAL PROPERTY MODEL (MPSI)

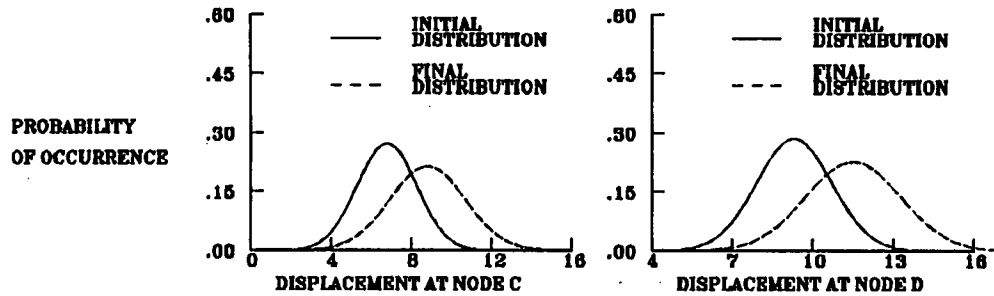


FIGURE (5) PROBABILISTIC DISPLACEMENTS CALCULATED BY NESSUS ( $10^{-3}$  INCH)

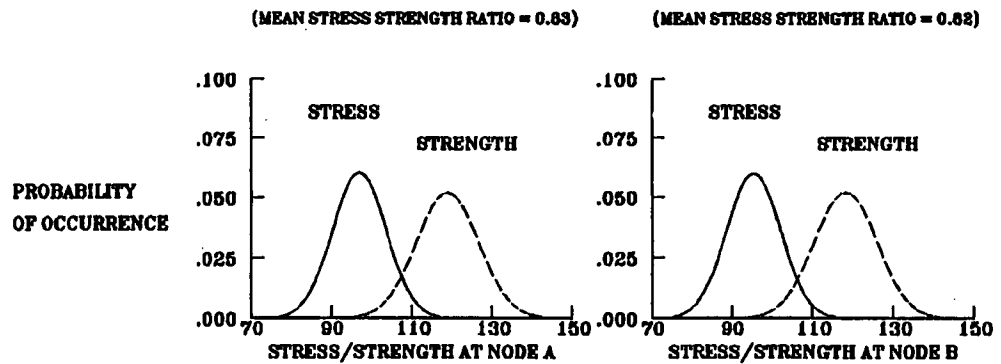


FIGURE (6) PROBABILISTIC FATIGUE STRESS/STRENGTH SIMULATED BY USING THE GENERIC PROBABILISTIC MATERIAL PROPERTY MODEL (KSI)

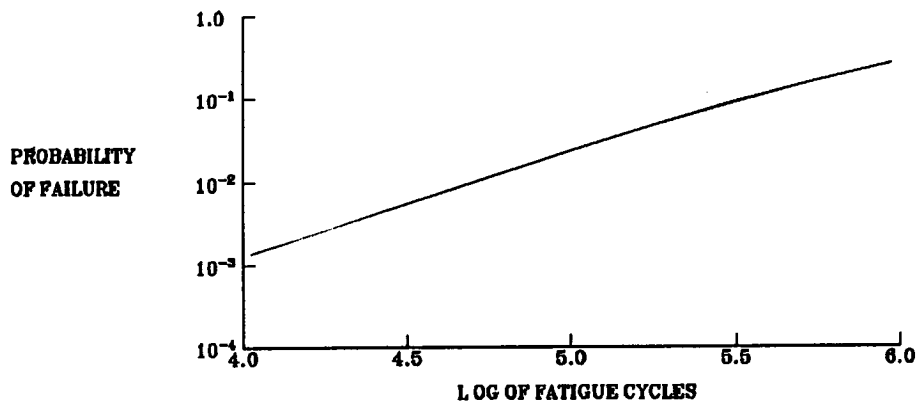


FIGURE (7) PROBABILITY OF LOCAL FAILURE DUE TO FATIGUE CYCLES AT NODE A

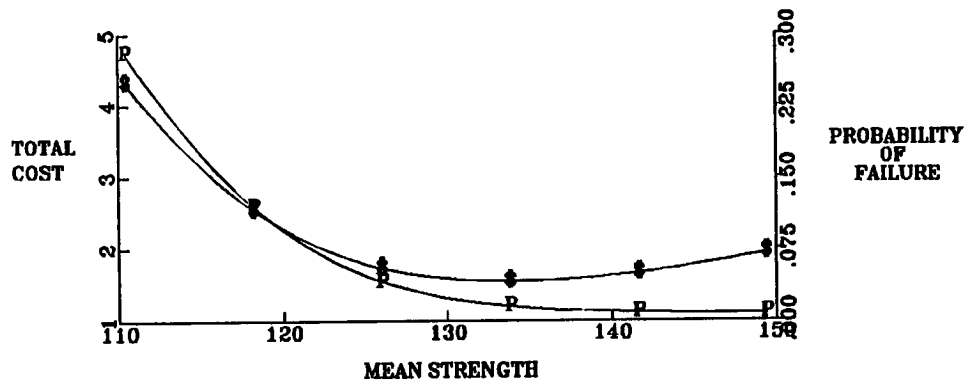


FIGURE (8) TOTAL COST AND STRUCTURAL RELIABILITY QUANTIFIED IN TERMS OF MEAN STRENGTH (GIVEN QUALITY)

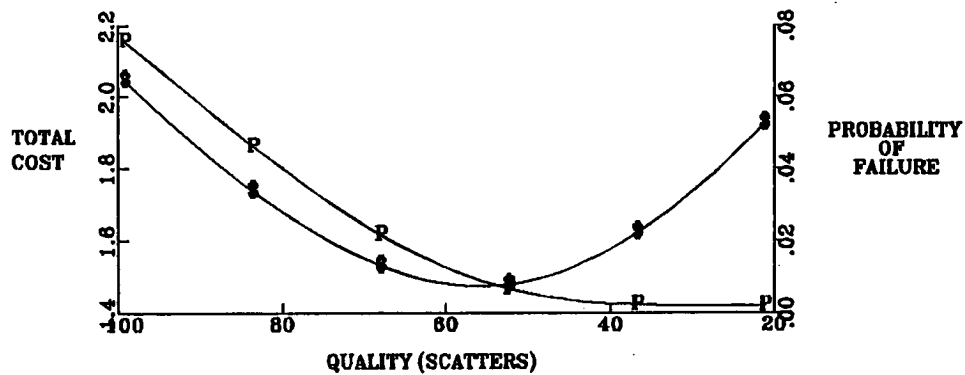


FIGURE (9) TOTAL COST AND STRUCTURAL RELIABILITY QUANTIFIED IN TERMS OF QUALITY CONTROL (GIVEN MEAN STRENGTH)

$$M_P = M_{P0} \left[ \frac{T_F - T}{T_F - T_0} \right]^n \left[ \frac{S_F - \sigma}{S_F - \sigma_0} \right]^p \left[ \frac{\log N_{MF} - \log N_M}{\log N_{MF} - \log N_{M0}} \right]^q$$

**PRIMITIVE VARIABLES**

$M_P$  = MATERIAL PROPERTY

$T$  = TEMPERATURE

$S$  = STRENGTH

$\sigma$  = STRESS

$N_M$  = MECHANICAL CYCLES

**SUBSCRIPTS**

$F$  = FINAL CHARACTERISTIC VALUE

$0$  = REFERENCE PROPERTY

**TABLE(1) GENERIC PROBABILISTIC MATERIAL PROPERTY MODEL  
IN TERMS OF PRIMITIVE VARIABLES**

RANDOM FIELDS	NUMBER OF DEPENDENT R.V.	MEAN	STANDARD DEVIATION (OR COV)	CORRELATION LENGTH	NUMBER OF INDEPENDENT R.V.
X COORDINATE	55	DETERMINISTIC COORDINATE	0.01 in	5.0	13
Y COORDINATE	55	DETERMINISTIC COORDINATE	0.01 in	5.0	13
Z COORDINATE	55	DETERMINISTIC COORDINATE	0.01 in	5.0	13
TEMPERATURE	55	STEADY STATE TEMPERATURE	60 °F	3.0	22
MODULUS	40	23 HKSI	0.10 (COV)	3.0	16
PRESSURE	36	STEADY STATE PRESSURE	0.20 (COV)	0.0	36
ROTATION SPEED	1	40000 RPH	0.01 (COV)	N/A	1

**TABLE (2) RANDOM INPUT DATA**

VARIABLE	DISTRIBUTION TYPE	MEAN	STANDARD DEVIATION (VALUE) (% OF MEAN)
$T_F$	NORMAL	2760 °F	51.4 °F 2.0
$T_0$	NORMAL	68 °F	2.04 °F 3.0
$S_F$	NORMAL	212.0 ksi	10.6 ksi 5.0
$\sigma_0$	CONSTANT	0	0 0
$N_{MF}$	LOGNORMAL	$10^8$	$5 \times 10^6$ 5.0
$N_{M0}$	LOGNORMAL	$10^3$	50 5.0
$n$	NORMAL	0.25	3.0
$p$	NORMAL	0.25	3.0
$q$	NORMAL	0.25	3.0

**TABLE(3) PRIMITIVE VARIABLE PROBABILITY DISTRIBUTIONS FOR  
PROBABILISTIC MATERIAL PROPERTY MODEL**

**ITERATION 0**

**STEP 1: CALCULATE THE PROBABILITY DISTRIBUTIONS OF MATERIAL PROPERTIES USING THE GENERIC MATERIAL MODEL WITHOUT STRESS AND FATIGUE CYCLES**

**STEP 2: CALCULATE THE NODAL STRESS BY NESSUS**

**ITERATION 1**

**STEP 1: CALCULATE THE PROBABILITY DISTRIBUTIONS OF MATERIAL PROPERTIES USING COMPLETE GENERIC MATERIAL MODEL**

**STEP 2: CALCULATE THE NODAL STRESS BY NESSUS WITH UPDATED MATERIAL PROPERTIES**

**ITERATION 2**

**(REPEAT THE ITERATIONS UNTIL THE PROBABILITY DISTRIBUTIONS OF MATERIAL PROPERTIES AND STRESS HAVE CONVERGED)**

**TABLE (4) PROBABILISTIC STRUCTURAL ANALYSIS BY NESSUS  
& A GENERIC PROBABILISTIC MATERIAL MODEL**